

Integration of complex calculators into Mathematics Education

Karmaşık Hesap Makinelerinin Matematik Eğitime Entegrasyonu

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ÖZET

Bu araştırma, TI-92 hesap makinesinin ortaöğretim seviyesine entegrasyonunu incelemek üzere yapılmıştır. TI-92 hesap makinesi bir sembolik hesap programı olan Derive ve bir geometri programı olan Cabri programlarıyla donatılmıştır. Bu çalışmada sadece Derive programı ile ilgili olan kısmı incelenmiştir. İlk olarak yeterliliklerini ve sınırlılıklarını belirledikten sonra, öğrencilerin öğrenme aşamasında, ne gibi « faydalarını » bekleyebiliriz sorusuna cevap arayacağız.

Anahtar Kelimeler: TI-92 hesap makinesi, enstrüman, Derive

ABSTRACT

This study was carried out to examine the integration of the calculator TI-92 in the secondary level. As a symbolic calculator, the TI-92 is equipped with Derive which is a symbolic calculation program and Cabri which is a geometry program. This study only dealt with the part concerned with the program Derive. First of all we determined the potentials and limitations, then we seek a response to the question which «benefits» can we expect at the learning level of the students?

Key words: Calculator TI-92, instrument, Derive

1. Introduction

As all human activities, mathematics is also dependent upon the culture in which it develops and the tools it uses for this purpose. Mathematics plays an important role in the scientific culture and information technologies today. It becomes an absolute necessary to employ these tools in teaching mathematics. The efficiency of these technologies in education can be determined by the measurement of its solution capability of the problems encountered in normal education. In our opinion the question “what sort of contribution can information technologies make in mathematics education” is not appropriate since this question underestimates the reality of this contribution. We would like to rephrase this question as “what does the mathematic education in current culture teach with information technologies”.

This study was carried out to investigate the integration TI-92 calculator, which is unique of its type in the current market, into secondary education. TI-92 calculator is equipped with symbolic calculus program *Derive* and a geometry program *Cabri*. We will first determine its limitations and capacities and try to find out its benefits in teaching mathematics.

The questions we will seek to answer are the following:

1. What are the potentialities of this calculator in the teaching of mathematics?
2. On the contrary we are likely to encounter which obstacles either material, cognitive, educational or institutional kind while using TI-92 in teaching mathematics?
3. What sort of uses can TI-92 have in facilitating learning process of the students?

On the theoretic plan, this sort of questioning may be based upon the approaches socio-cultural and anthropological which play an important role in mathematics education and appear for at least today as complementary of constructivist approach which is dominant for a long time (Chevallard, 1992).

2. What is an instrument?

An example: TI-92 calculator. It is described as a technical tool. However it may not have the same meaning for the two people arbitrarily chosen. The opinion you have may largely be dependent upon the part of the tool chosen and your working habits with this tool.

The opinion which appears as a cognitive psychology enables us to explain the objects in a much better way. Rabardel (1995) made the following distinction:

1. Technical object: it is necessary to talk about the artefact, then it is an objective concept.
2. Instrument, defined by the association of a part of the artefact chosen by the subject, and by the usage schemes constituted by the subject when he adapts himself to the technical tool.

It can be represented schematically as depicted Figure 1:

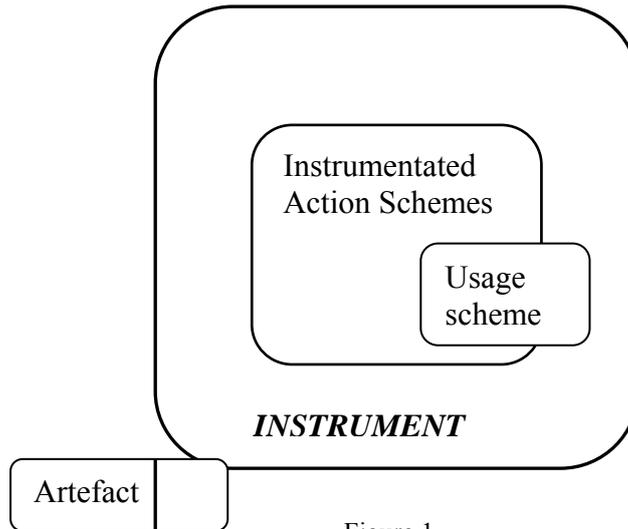


Figure 1

It is necessary to clarify the words and terms related with the scheme. Rabardel mentions a “nomad concept” related to it. We will make a description similar to that of Vergnaud (1991): A scheme is the invariant organisation of the behaviour for a given

class of events. For example, a solution scheme of a second degree equation or a scheme to find a solution for limit in a calculator.

Vergnaud (1991), specifies that a scheme has a signification who in any way surpasses it: a representation implicit or explicit of the real makes integrative part of the scheme, analysable as invariant operatives.

Starting from the use of an instrument, we will distinguish usega schemes (e.g finding an approximate value for a number having an exact value) and instrumentated action schemes (e.g searching for a window which correctly pictures the garaphical representation a function being investigated). As we have shown in above scheme all the instrumentated action schemes contain certain number of usage schemes.

Therefore an instrument can be described as a construction independent of the individual (a technical object or an artificial tool) or a construction unique to the person.

In these unique constructions (Rabardel (1995) describes this as “instrumentational formation”) there occurs two common processes; Instrumentation and instrumentalisation. Instrumentation is described as the enrichment of the artificial tool by person while instrumentalisation means making the concepts of the person appropriate.

In more simple terms in instrumentation the person changes the tool and in instrumentalisation the tool changes the person...

Instrumentation and instrumentalisation are also the self construction of the person and results as an integration of the people or the organizations (a firme, an educational institute or parental institute) who introduces the artificial tool, into the socialisation process.

The roles of the artificial tools into the activities of the individulas are based upon two dimensions:

- The problems created by the artificial tool and their effects

- The opportunities by these tools presented as a source, working type, organisation styles.

The calculator TI-92 we are interested in this study provides diversified possibilities to the student such as making differentiation, calculation of limit or obtaining graphical representation with an incomparable power and fastness. However the efficient use of this machine requires strict compliance to the regulations of the producer and in depth evaluation of error messages or unexpected results. The students are likely to encounter various problems when using this calculator.

The transfer process in other words the transformation of an informative representation from the mathematical representation is named as “cognitive transformation” (Balacheff, 1994).

3. The Capacities of TI-92 Calculators

The literature concerning usage of TI-92 calculators in education assign to these calculators some potentialities for teaching and learning.

These potentialities concern also the nature of mathematic activities which these environments propose to students.

Usage of calculators in education is presented often as below.

- It enables to develop more efficiently an experimental approach of mathematics with its phases of exploration, distribution and test of conjectures, evaluation as well. This affirmation is especially discussed appealing the facility of using this environment, strongly reduced cost of exploration activities when compared with traditional environments and also some characteristics of student/machine interaction (Hirlimann, 1994).
- It enables to reach more rich, more interesting, less purely scholastic problems than ordinarily met problems because of the various helps in the resolution (numeric, algebraic, graphic) provided by the machine.

- It enables a better conviviality of education and a better adaptation of it to particularity of every student's performance. This aspect is classically discussed assigning the characteristics of communication student/machine and comparing them with the characteristics of communication student/teacher. Generally, in these environments, error takes a status less penalizing and the teacher, unburdened by the machine from certain number of mediations and assessments, is more available for the problems which really needs the involvement of the teacher and he is perceived as a person resource and a partenaire in the resolution of the tasks proposes (Kutzler,1994).

It is clear that these different potentialities are generally perceived as leading an accession of the students' motivation for the discipline and an accession of the activity in the lesson.

Usage of calculators is also presented, in a plan more cognitive, as:

- It enables until a point to correct the difficulties mathematics encountered by some students and allow them to continue learning without any interruption with the same obstacle. Concerning the software *Derive*, it is invoqued especially the possibility of correcting the difficulties in the domain of calcul numeric and elementary algebra (Canet, 1994).
- It favorises students releasing from calculs and operations of graph drawing, briefly from technical studies and it provides a functioning more reflexive, more strategic, more conceptual and in consequence understanding, sense, control more than acquisition of simple capacities of execution technics. This point is without doubt one of the plus frequently cited and associated, generally, to an ordinary education, is introduced as focused on acquisition of the pure technic competences (Forster, 2004).
- It favorises, by the possibilities of visualisation, the development of mental images and that's why, the comprehension mathematics and also by the performance conjoint in numeric, algebric and graphic representations, a better articulation of these representations in mathematics studies (Aldon, 1994).

- In algebra it favors the studies on the syntactical aspects, the recognition of the forms, on the status of different manipulated objects, allowing them to live as demand of environment (Artigue, Defouad, Dupérier, Juge, Lagrange, 1998).
- It enables to modify the relationship of students between numbers, by the possibility of executing exact calculations. In a plan cognitive, this aspect is especially perceived as responsive of making some misconceptions of numbers (Artigue, Lagrange, 1999, May).

4. Sufficiencies and Limitations of TI-92 Calculators

TI-92, as other symbolic calculation tools, enables to reach immediately to various representations of a mathematics object. For example, one will get very easily various scripts of an algebraic expression, one can choose construct a figure of geometry with or without usage of a coordinate system. This tool facilitates the transition between representations calling different registries. Especially, the geometry application includes important novelties in relation to CABRI. As Capponi (1995) indicated, it enables to connect the aspect numerics and geometrics and to represent graphically of phenomenon in geometric base.

Therefore, it may realize some exact numeric calculations and some symbolic calculations (derivation, limit, integration,...). However, it may also make approximate calculations: the graphic representation of a function for example. Making a distinction between an exact calculation and an approximate calculation is not an easy task, neither for the inventor of the machine nor for the user.

4.1. The relation between the exact and approximate calculations

We are going to deal here some radically new concepts such as the relation between the exact and and approximate calculations.

The distinction between the the exact and approximate values is obvious. The “mode” menu enables us to choose the type of calculation we wanted to use (Figure 2): the options are exact, approximare and auto. The use of first two is clear. But if we want to

use the third option we obtain sometimes the approximate and sometimes the exact values.



Figure 2

For instance as given in the following screen the calculator gives the exact values for π , $\sin(2)$, $\sqrt{2}$ but an approximate value for $2^{0.5}$ in “auto” mode (Figure 3). Same number calculation at the same mode but two different results.

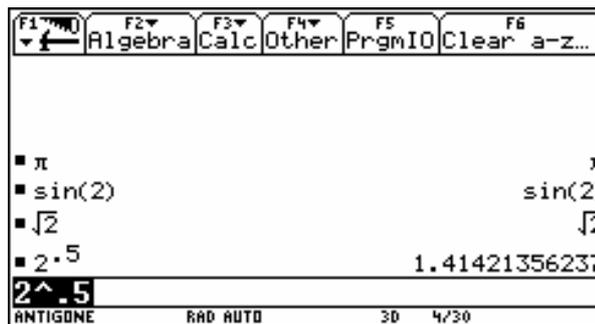


Figure 3

Let examine the relation of the approximate and exact values of the calculators which the caller seems to fully differentiate. For instance if we want the approximate value of π we obtain 3,141542653589 and if want the exact value we obtain π .

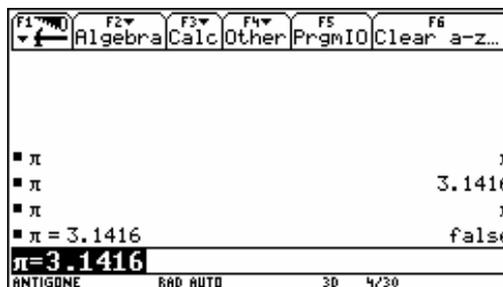


Figure 4

If we want the truth value of « $\pi = 3,1416$ » equation the calculation separates the exact value from the approximate value from each other and gives the answer as “false” (Figure 4).

However the control of exact values inevitably causes some loss of information. Since the numeric memory of the calculator is limited, it can not control more than 14 digits. Let see some of its consequences:

- The calculator shows that $1+10^{-13}$ and 1,0000000000001 are equal in exact mode
- On the other hand although the calculator recognises $1+10^{-14}$ it can not differentiate between 1 and 1,0000000000001 (in spite of being in exact mode) (Figure 5).

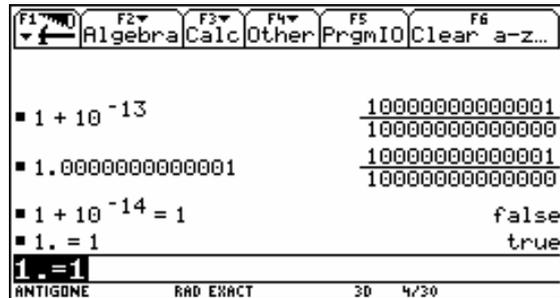


Figure 5

Its capacity to recognize the symbolic statements is not unlimited. The program is equipped with symbolic statement and routes which enable the transformation from one into another. This limited memory of the calculator may cause some surprises in “exact” calculation administration.

These limitations may result in different answers according to the calculation mode selected. In the calculation of the same number with different modes we obtain the following answers: (Figure 6).

- With exact mode «over capacity»
- With approximate mode «non- definitive »
- With automatic mode “ 0”.

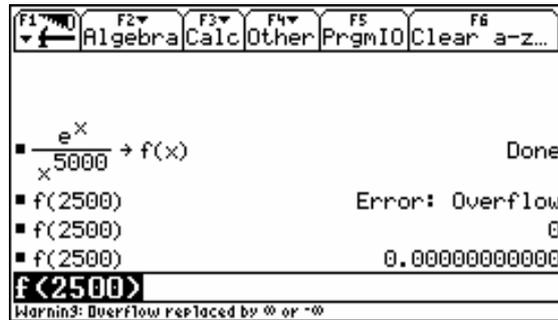


Figure 6

4.2 The presence of two different calculation modes may result in the propensity to escape from exact calculation.

This may be the case in limit calculation. If the program does not respond in exact mode we can carry out the investigation in approximate mode.

Lets take the example of the limit $f(x) = (\cos x)^x$ function at $\frac{\pi}{2}$. In exact calculation the limit is non definitive (It is clear that we have look at the limit from the left here) With two step escape route we can find the limit sought:

- The calculation of $f\left(\frac{\pi}{2}\right)$ in exact mode
- The use of “limit” command in approximate mode (the program gives the answer which rejects to give in exact mode) (Figure 7).

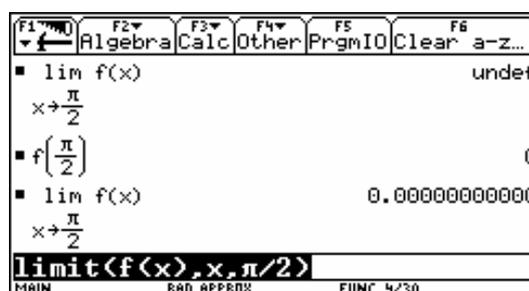


Figure 7

These escape routes are especially useful in solving equations. The *Derive* program in TI-92 calculator is capable of solving first degree and second degree equation in exact mode and provided that the roots are simple (or rational) it can solve higher degree equations. For other type of higher degree equations the program will give the answer in exact mode as the rearranged version of the equation. This will result the increased propensity of seeking solutions in approximate mode. The program will start a searching algorithm within a certain range

4.3 The order of words is much more important for symbolic calculators than the graphical calculators.

There are two possibilities here as in the case of graphical calculators: we can activate a command from either the graphical applications or start menu.

- *Mathematical commands in graphical applications (Figure 8)*

It always carries out calculations in approximate mode.

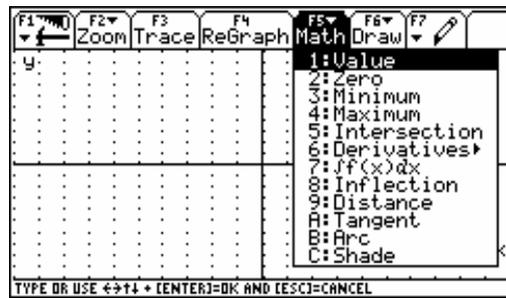


Figure 8

- The commands of algebra menu in the start menu (Figure 9)

These commands can be used in exact, automatic or approximate modes.



Figure 9

- The commands of "calculus" menu in the start menu (Figure 10)

We can use these commands in all calculation modes.



Figure 10

In graph plotting calculators we have to choose a command from the menu while we can directly write a command in the start line in symbolic calculators. The memorisation of the symbols can speed up the calculations. For instance in order to obtain a limit of f at a point of a we must write «limit($f(x),x,a$)» If we write «limite($f(x),x,a$)» the calculator will kindly reject us. There are three types of writing in mathematics.

- In English «limit»
- In Mathematics «lim»
- In Derive program «limit»

The word order necessity has a constructive character. We have to be able to distinguish between the concepts such as function and parameter. We can hypothesise that the symbol “dt” which signifies the differential can hardly be forgotten by the students. Here this element is important to separate the variables and parameters (Şekil 11).

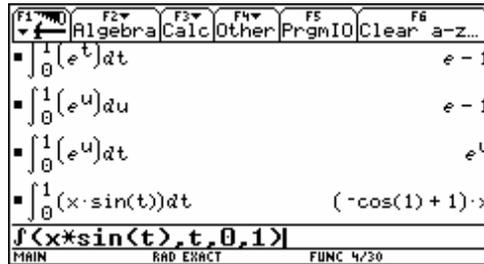


Figure 11

The free choice of letters requires the use of a process for the separation of the functions of the variables. For instance if we write $\sin x$ the program will take this chain of characters as a new variable. In order to determine the sinus of x we have to write $\sin(x)$.

Similarly ax will represent a variable. If we want to show a product of a and x we have to put multiplication sign between these two letters.

5. Results and Suggestions

The discussion about the use of calculators in the classroom, exams and university entrance exams doesn't finish. The students may use the calculators in order to find the solution of the problems, to verify their answers. The variety of such calculators in the market will provide an unfair advantage to the students who know the correct use of these machines.

The use of a calculator is not an easy thing and requires a specific learning process. Its usage without mastering may cause hardly on the construction of mathematics knowledges. It must be considered by the teacher.

In education, the presence of the calculators may enable to dedicate more of the time to the question of choosing the representation because of partially releasing from the task of production. One can, for example, envisage usage at the end of the 8th grade for make understand along the examples the reasons which lead to choose that writing in a given situation, make recognize a mathematics object under different sophistications. These are in fact the competencies that the teachers search for developing in their students. Remember that at the phase of conceptualization the conceptual level and technics are not independent: the situations must not totally consist of technic studies.

Such tools may enable us to construct some situations where the mathematics takes reality scientific among students. This may pose a question about their confidence to their calculators and on the other hand this will bring them to a personal reflection. The teacher's objective can not be protecting the students from all confrontations with mathematics.

The intelligent use of such a tool, which demands a minimum understanding of these mechanisms, results from the competencies that we try to achieve our students. The teacher must evoke the reflection about the produced representations of mathematics objects, originally from the calculator or from a study pencil/paper.

It is essential, if we want really help an effective integration of computing tools in education, to be convinced of the potential benefits of this integration, but a conviction without lucidity will not help to overcome the obstacles.

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